Between MDPs and semi-MDPs: A framework for temporal abstraction in reinforcement learning

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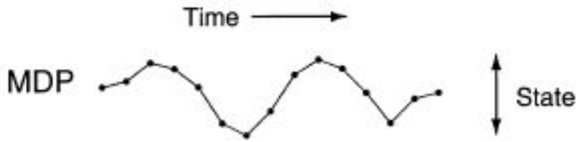
Motivation

- Learning, planning, and representing knowledge at **multiple levels of temporal abstraction** are longstanding challenges for AI
- Many real-world decision-making problems admit hierarchical temporal structures
    - Example: planning for a trip
    - Enable simple and efficient planning
- This paper: how to automate the ability to plan and work flexibly with multiple time scales?

This paper

- Temporal abstraction within the framework of RL and MDP using **options**
  - Enable **temporally extended actions** and planning with **temporally abstract knowledge**
- Benefits
  - MDPs + options = semi-MDPs: standard results for SMDPs apply!
  - Knowledge transfer: use domain knowledge to define options, solutions to sub-goals can be reused
  - Possibly more efficient learning and planning

MDPs



- At each time step $t = 0, 1, ...$
    - Perceive state of environment $s_t \in S$
    - Select an action $a_t \in A$
    - One-step state-transition probability $p_{s,s'} = P(s_{t+1} = s' | s_t = s, a_t = a)$
    - At $t+1$, receive reward r_{t+1} and observe the new state s_{t+1}
- The goal is to learn a Markov policy $\pi : S \times A \rightarrow [0,1]$ that maximizes the expected discounted future rewards from each state:

$$V^{\pi}(s) = E[r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + ... | s_t = s, \pi]$$

Semi-MDPs

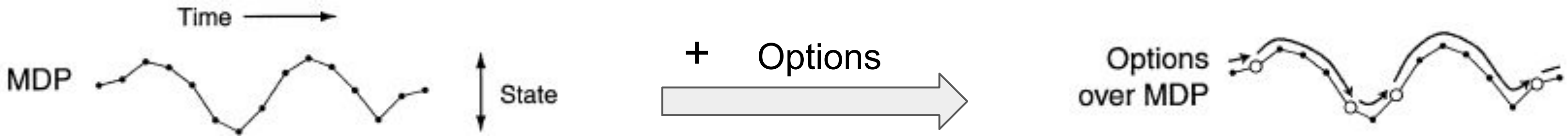
- State transitions and control selections at discrete times, but the time between successive control choices is variable
- Allows for temporally extended courses of actions and Markovian at the level of decision points
- However, temporally extended actions are treated as indivisible and unknown units

Options

- Goal: generalize primitive actions to include temporally extended courses of actions with internally divisible units
- An **option** (I, π, β) has three components:
    - A policy $\pi : S \times A \to [0, 1]$
    - A termination condition $\beta : S^{+} \to [0, 1]$
    - An initiation set $I \subseteq S$
- If option (I, π, β) is taken at $s \in I$, then actions are selected according to π until the option terminates stochastically according to β
- **Markov option**: within an option, policies and termination conditions depend on the current state
- **Semi-Markov option**: policies and termination conditions may depend on all prior event since the option was initiated

MDP + Options = Semi-MDP!

- **Theorem**: For any MDP and any set of options defined on that MDP, the decision process that selects only among those options and executing each to termination is an semi-MDP



- Implications:
  - This relationship among MDPs, options, and semi-MDPs provides a basis for the theory of planning and learning methods with options
  - i.e. MDPs + Options are more flexible compared to conventional semi-MDP, but standard results for semi-MDPs can be applied to analyze MDPs with options

Semi-MDP Dynamics

Semi-MDP Dynamics

- From $\mathcal{A}$ to $\mathcal{O}$

Semi-MDP Dynamics

- From $\mathcal{A}$ to $\mathcal{O}$
- From one-step to (stochastic) *k*-step

Semi-MDP Dynamics

- From $\mathcal{A}$ to $\mathcal{O}$
- From one-step to (stochastic) *k*-step

$$r_s^o = E\left\{r_{t+1} + \gamma r_{t+2} + \cdots + \gamma^{k-1} r_{t+k} \,\middle|\, \mathcal{E}(o, s, t)\right\}$$

$$p_{ss'}^o = \sum_{k=1}^{\infty} p(s', k)\gamma^k$$

Semi-MDP Infrastructure - this looks familiar...

$$V^{\mu}(s) = E\left\{r_{t+1} + \cdots + \gamma^{k-1} r_{t+k} + \gamma^{k} V^{\mu}(s_{t+k}) \,\middle|\, \mathcal{E}(\mu, s, t)\right\}$$

(where k is the duration of the first option selected by μ)

$$= \sum_{o \in \mathcal{O}_s} \mu(s, o) \left[r_s^o + \sum_{s'} p_{ss'}^o V^{\mu}(s') \right],$$

Semi-MDP Infrastructure - this looks familiar...

$$V^{\mu}(s) = E\{r_{t+1} + \cdots + \gamma^{k-1} r_{t+k} + \gamma^{k} V^{\mu}(s_{t+k}) \mid \mathcal{E}(\mu, s, t)\}$$

(where k is the duration of the first option selected by μ)

$$= \sum_{o \in \mathcal{O}_s} \mu(s, o) \left[r_s^o + \sum_{s'} p_{ss'}^o V^{\mu}(s') \right],$$

$$V_{\mathcal{O}}^{*}(s) \stackrel{\text{def}}{=} \max_{o \in \mathcal{O}_s} \left[r_s^o + \sum_{s'} p_{ss'}^o V_{\mathcal{O}}^{*}(s') \right]$$

Semi-MDP Infrastructure - this looks familiar...

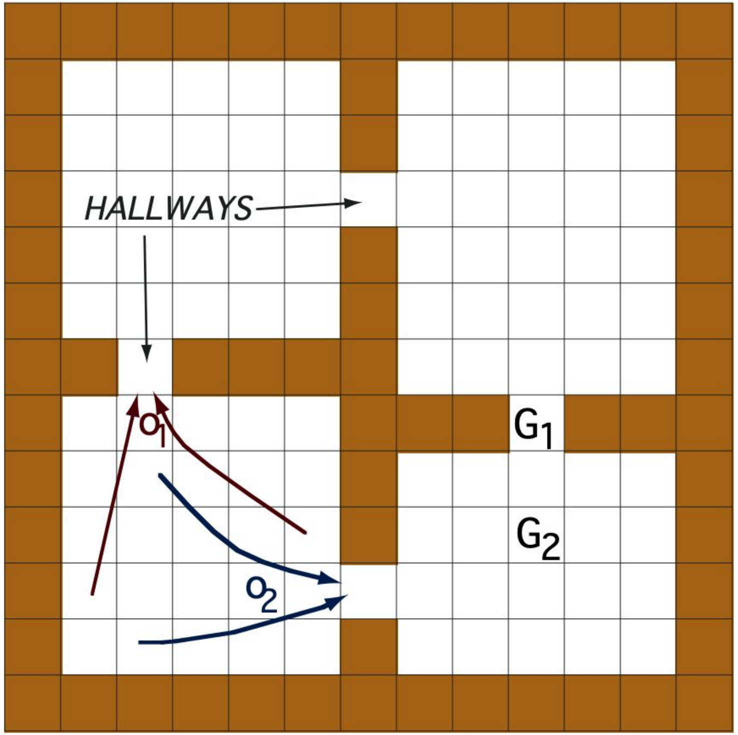
$$V^{\mu}(s) = E\{r_{t+1} + \cdots + \gamma^{k-1} r_{t+k} + \gamma^k V^{\mu}(s_{t+k}) \mid \mathcal{E}(\mu, s, t)\}$$

(where k is the duration of the first option selected by μ)

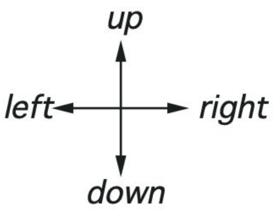
$$= \sum_{o \in \mathcal{O}_s} \mu(s, o) \left[r_s^o + \sum_{s'} p_{ss'}^o V^{\mu}(s') \right],$$

$$V_{\mathcal{O}}^*(s) \stackrel{\text{def}}{=} \max_{o \in \mathcal{O}_s} \left[r_s^o + \sum_{s'} p_{ss'}^o V_{\mathcal{O}}^*(s') \right]$$

Allows for planning & learning analogously to in MDPs!



4 stochastic
primitive actions

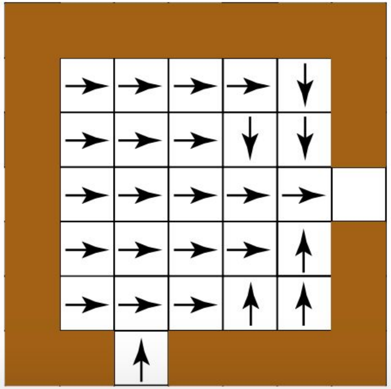


Fail 33%
of the time

8 multi-step options

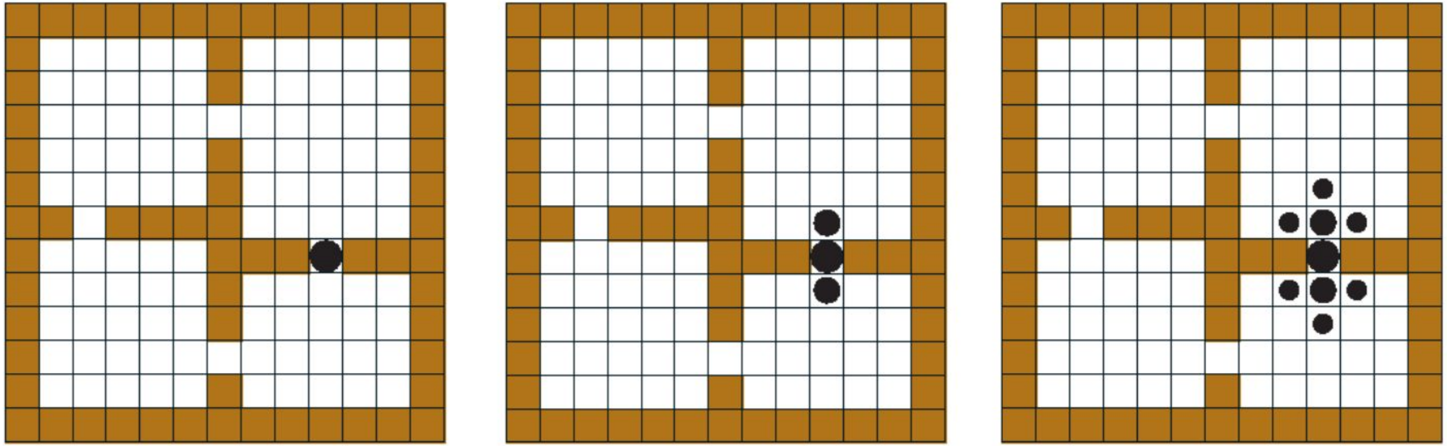
(to each room's 2 hallways)

Example of one option's policy:

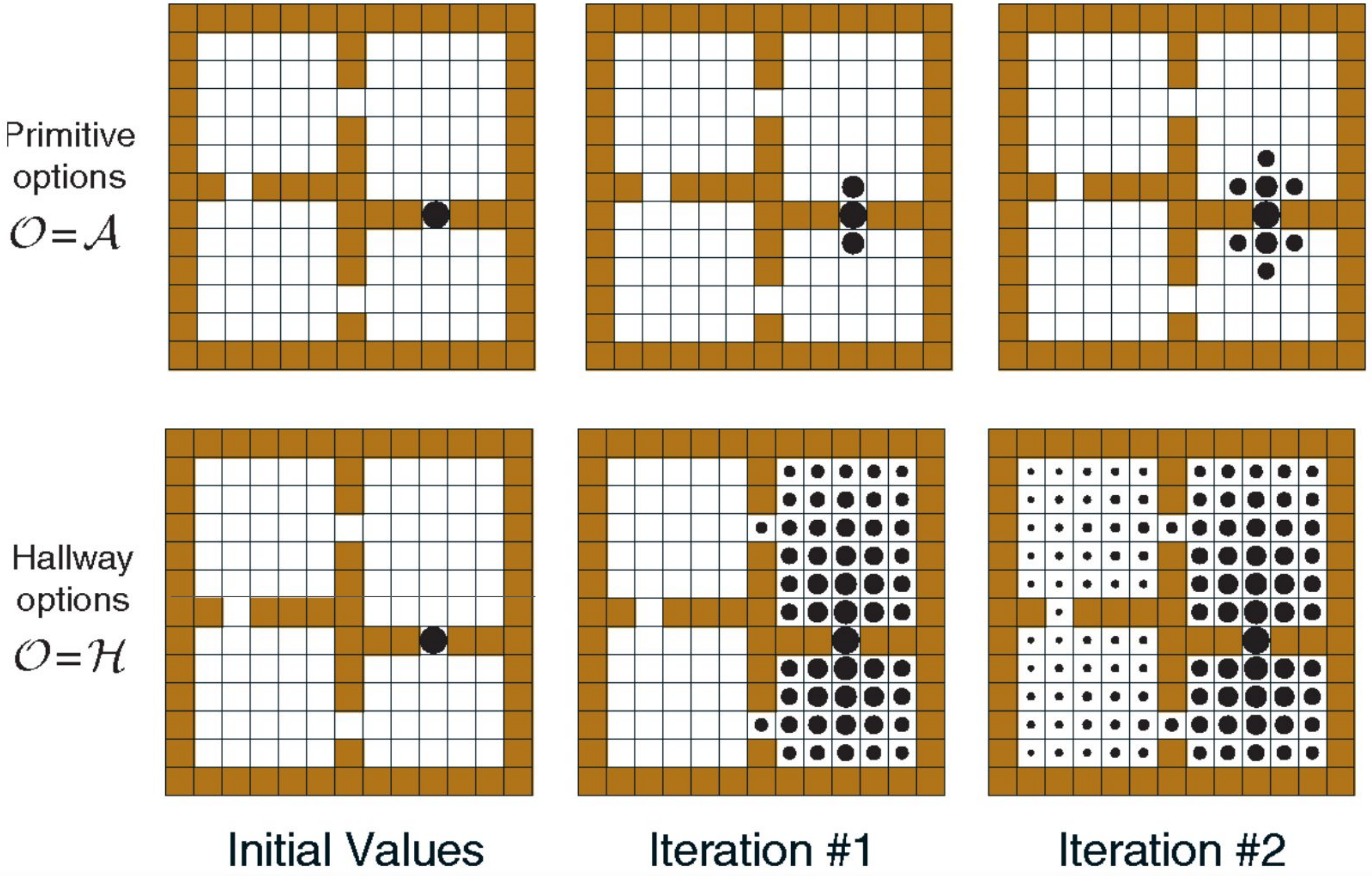


Target
Hallway

Primitive options
$\mathcal{O}=\mathcal{A}$

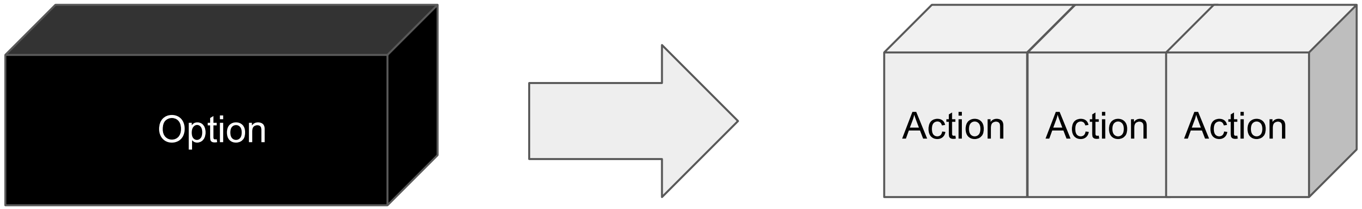


Initial Values　　Iteration #1　　Iteration #2


Primitive options
$\mathcal{O}=\mathcal{A}$
Hallway options
$\mathcal{O}=\mathcal{H}$
Initial Values
Iteration #1
Iteration #2

Between MDPs and Semi-MDPs...

Open up the black-box when
Option is Markov!



- Interrupting options
- Intra-option model / value learning
- Subgoals

I. Interrupting options

- Don’t have to follow options to termination!
- At time t, if continue with o:

$$Q^{\mu}(s_t, o)$$

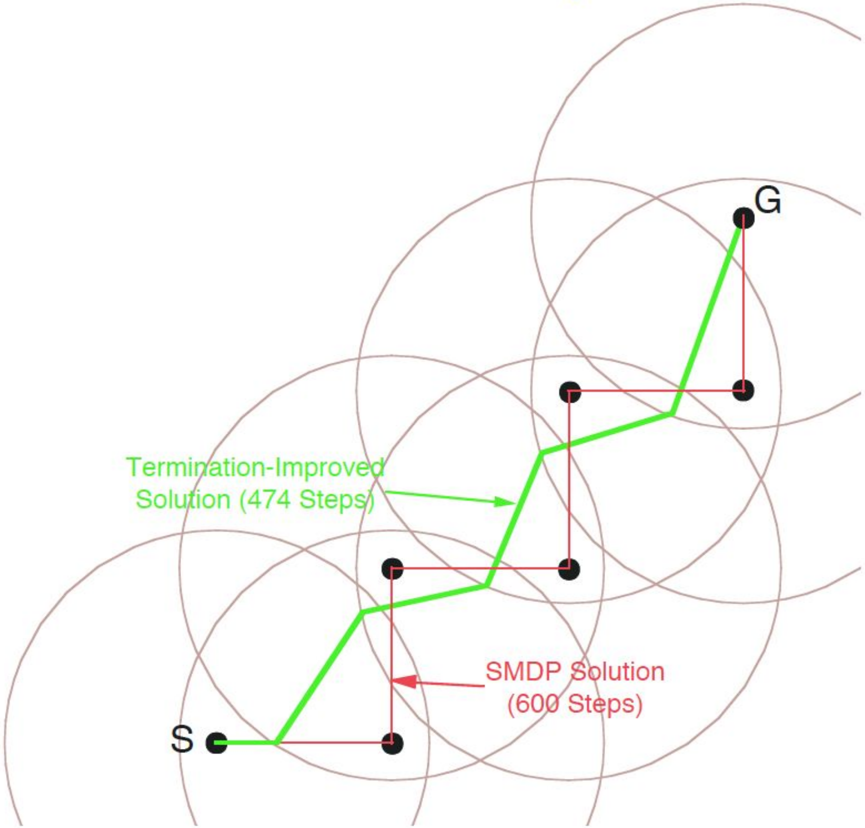
If select new option:

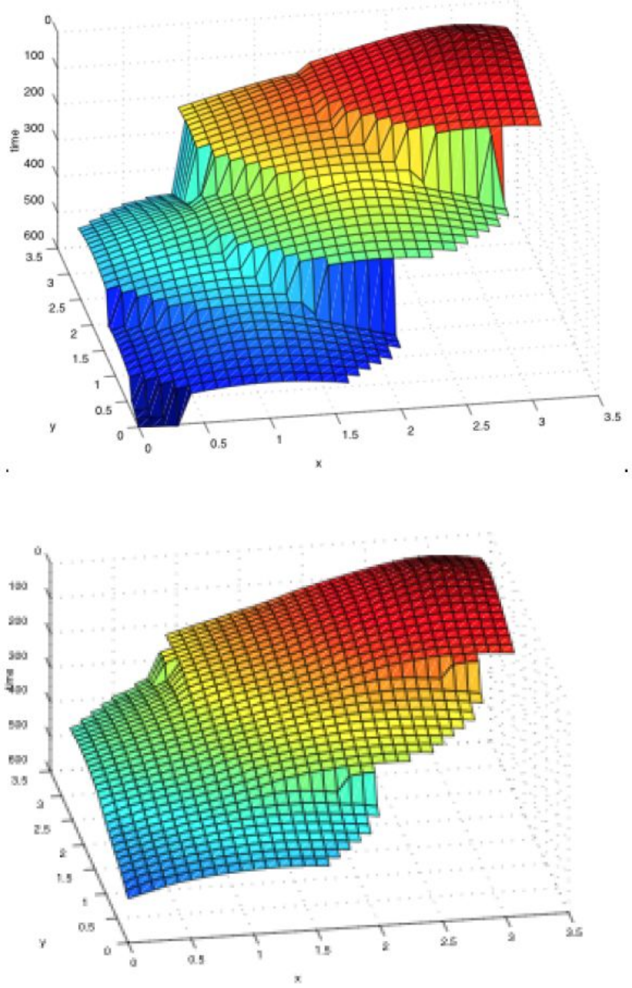
$$V^{\mu}(s_t) = \sum_{o'} \mu(s_t, o') Q^{\mu}(s_t, o')$$

- Policy $\mu \to \mu'$ Interrupted Policy

- For all s, $$V^{\mu'}(s) \geq V^{\mu}(s)$$

Landmark example





II. Intra-option **model** learning

$$\text{Given } o = (I, \pi, \beta), \text{ learn model } r^o_s,\ p^o_{s,s'}.$$

Intra-option **value** learning

$$\text{Given } o = (I, \pi, \beta),\ r^o_s,\ p^o_{s,s'}, \text{ learn value function } Q^*_{\mathcal{O}}(s, o).$$

- Take an action, update estimates for all **consistent** options.

SMDP-Learning vs. Intra-option Learning

SMDP	Intra-option Learning
Update only when option terminates	Update after each action (Learn from fragments of experience)
Update 1 option at a time	Update all options consistent with current action (off-policy, can learn never-selected options)
Semi-Markov options	Only Markov options

III. Learning options for subgoals

- Can we learn the policy that determines an option?
  - Yes: add terminal subgoal rewards
  - Perform Q-learning to adapt policies towards achieving subgoals
  - Subgoals + rewards must still be given

Conclusion

- Strengths
  - General framework for reinforcement learning at different levels of temporal abstraction
  - Mimics real-world setting of sub-tasks and sub-goals
  - Same formulations and algorithms apply across levels
  - “Efficiency” in planning
- Weaknesses
  - Domain knowledge required to formalize options/subgoals
  - Options may not generalize well across environments
  - Might necessitate a small state-action space

Questions + Discussion

- How does the temporal abstraction framework relate to meta-learning?
- Can you imagine environments for which this framework cannot be applied in a straightforward way, or for which adopting this framework might be disadvantageous?
  - What if the state that we observe is a noisy version of the actual state? Are options still useful in the partially-observable setting?
- Hierarchical abstraction for both state space and action space?
- Possible extensions for intra-option learning:
  - Use **reweighting** to learn about **inconsistent** options?
  - Concept of **consistency** between option and action for **stochastic** options?